University College Dublin
An Coláiste Ollscoile, Baile Átha Cliath

SEMESTER 1 EXAMINATION 2017/2018

MATH00030

Access to Science, Engineering and Agriculture: Mathematics 1

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Time Allowed: 2 hours

Instructions and Notes for Candidates

Candidates should attempt all questions.
Not all questions are allocated the same number of marks.
The exam is marked out of 100 marks.

Notes for Invigilators

Non programmable calculators are permitted.
The formula sheet provided is permitted.

1. (a) Without using a calculator, calculate the following.

Note that you should show enough of your working to demonstrate that you have not simply entered the expression into a calculator.

(i) $\frac{3}{5} - \frac{4}{3}$

(ii) $\frac{3}{5} \times \left(-\frac{7}{11}\right)$

(iii) $\frac{2}{7} \div \frac{9}{13}$

(iv) -2^2

(v) $\left(\frac{27}{64}\right)^{-\frac{2}{3}}$

(vi) $(3 \times (-4) - 5 \div (-6))^2$

(vii) $\log_5 125$

(viii) $\log_8 \frac{1}{2}$ **[8]**

(b) Simplify the following expressions by expressing them as a single power of x.

(i) $x^{-7} \times x^4$

(ii) $x^{-\frac{1}{3}} \div x^{\frac{2}{5}}$

(iii) $(x^{-3})^2$ **[3]**

(c) Express $\log_a \left(\left(\frac{x^2}{y^3}\right)^{-3}\right)$ in terms of $\log_a x$ and $\log_a y$ **[2]**

(d) (i) Approximate 9.99999 to three decimal places.

(ii) Approximate 0.0004499 to one significant figure.

(iii) Express 0.000456274 in scientific notation.

(iv) Express 24901624 in scientific notation to two significant figures. **[4]**

(e) Simplify $(-x^2 + 2x - 2) - (x^2 + 4x - 4)$ **[1]**

(f) Multiply out $(2x^4 - 4x)(-x^2 + 2)$ **[2]**

(g) Perform long division on $\frac{x^2 - 3x - 1}{x - 2}$, giving the quotient and remainder. **[4]**

(h) Expand $\sum_{i=-2}^{2} -ix^i$ **[2]**

(i) Calculate $\binom{10}{8}$ without using a calculator.

Note that you should show enough of your working to demonstrate that you have not simply entered the expression into a calculator. **[2]**

(j) Expand $(2x + 3y^2)^3$ using The Binomial Theorem. **[4]**

2. (a) Find the equation of the line through the point $(-1, -3)$ parallel to the line $y = 2x + 1$. **[2]**

(b) Solve the simultaneous equations

$$2x - 5y = 18$$
$$-3x - 4y = -4$$

[3]

(c) Find the length of the line segment between the points $(-2, -3)$ and $(-4, 2)$. **[1]**

3. (a) Write the expression $3x^2 - 4x + 1$ in completed square form. **[3]**

(b) Solve the equation $3x^2 - 4x + 1 = 0$ by using the completed square form you found in Part (a). **[2]**

(c) Sketch the graph of the function $y = 3x^2 - 4x + 1$, showing the y-intercept, the x-intercept(s) (if applicable) and the turning point. **[4]**

4. (a) For each of the following:

- Say whether or not it is a function and if not say why not.
- If it is a function state the domain and the codomain.

(i)

$$f : \mathbb{R}^+ \to \mathbb{R}^-$$
$$x \mapsto -2x - 1$$

(ii)

$$f : \mathbb{R}^- \to \mathbb{R}$$
$$x \mapsto \sqrt{x} + 1$$

[4]

(b) Sketch the graph of the function

$$f : \{x \in \mathbb{R} : \ -2 \leqslant x \leqslant 2\} \to \{x \in \mathbb{R} : \ -10 \leqslant x \leqslant 10\}$$
$$x \mapsto 3x - 1$$

[2]

(c) For each of the following functions, say whether or not they are injective, surjective or bijective. If a function is not injective, surjective or bijective then say why not.

(i)

$$\begin{aligned} f\colon \{1,2,3,4\} &\to \{A,B,C,D\} \\ 1 &\mapsto A \\ 2 &\mapsto B \\ 3 &\mapsto D \\ 4 &\mapsto B \end{aligned}$$

(ii)

$$\begin{aligned} f\colon \mathbb{R} &\to \mathbb{R} \\ x &\mapsto 2x-3 \end{aligned}$$

[3]

(d) For each of the bijective functions you found in Part (c), give the inverse function. **[2]**

(e) Solve the equation $9^{2x} = 8$. **[3]**

5. (a) Convert $135°$ to radians, leaving your answer as a multiple of π. **[1]**

(b) Convert $\dfrac{5\pi}{3}$ radians to degrees. **[1]**

(c) Using the geometric method, find $\cos\left(\dfrac{7\pi}{6}\right)$ without using a calculator. **[3]**

(d) Find the size of the angle A in the triangle in Figure 1.
You should give your answer correct to two decimal places **[3]**

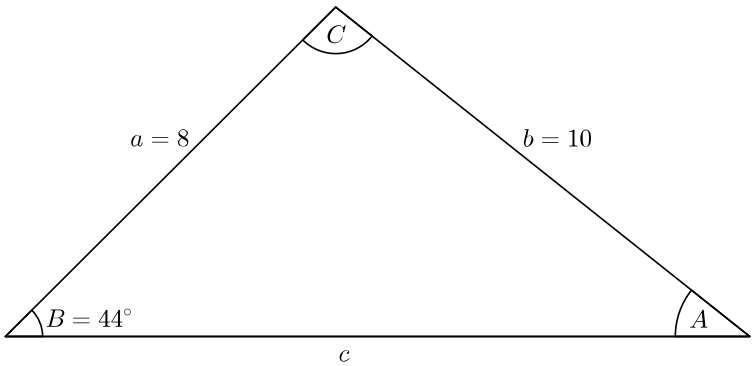

Figure 1: The triangle for Question 5 (d).

(e) Using whichever trigonometric formulae you like, but without using a calculator, calculate the following.

Note that you should show enough of your working to demonstrate that you have not simply entered the expression into a calculator.

(i) $\cos\left(\frac{7\pi}{12}\right)$

(ii) $\tan\left(\frac{\pi}{8}\right)$ **[4]**

6. (a) Find the derivative of $f(x) = -2x + 5$ using first principles. **[2]**

(b) Find the derivatives of the following functions.

(i) $f(x) = x^3$

(ii) $f(x) = \sin(-4x)$

(iii) $f(x) = \cos\left(\frac{1}{3}x\right)$

(iv) $f(x) = -3x^{-\frac{1}{3}} + 2e^{-2x} - 5\ln(-5x)$ (where $x < 0$) **[6]**

7. Find the following integrals.

(a) $\int 2\,dx$ **[1]**

(b) $\int_{-1}^{1} x^3\,dx$ **[2]**

(c) $\int_{0}^{\frac{\pi}{6}} \cos(3x)\,dx$ **[2]**

(d) $\int 2e^{-4x} - 3x^{-1}\,dx$ **[2]**

8. (a) For the list of numbers $-3, -4, -9, -2, -5, -6, -6, -7$, find the

(i) Mean

(ii) Median

(iii) Mode(s)

(iv) Interquartile range **[5]**

(b) Find the line of best fit using the least squares method with the points $(-4, 3), (-2, 0), (0, 1), (2, -2)$ and $(5, -6)$. **[7]**

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